3.6

Mathematics of Finance

Interest Compounded Annually:

If a principal P is invested at a fixed annual interest rate r, calculated at the end of each year, then the value of the investment after n years is:

$$A = P(1+r)^n \quad \text{(r is a decimal)}$$

Ex. Suppose Jessie finds a $51 bill on the ground at Utah State. She decides to put it in the bank until she graduates. Many obstacles prevent her from graduating for several years. Twenty-five years later at her graduation she finds a $3 bill and remembers the $51 she put in the bank. How much money does Jessie now have if the rate was 11.5%?

$A = 51(1+.115)^{25} = \$775.25$ +$3 that she found = $778.25!

Compounding k-times per year

$A = P(1+\frac{r}{k})^{kt}$ k is the number of times per year interest is compounded

EX. Jessie has become a little wiser now that she has finally graduated from college. She decides to put her $778.25 in another account with an interest rate of 13.2% that compounds monthly. How much money will she have after she receives her master's degree 10 years later?

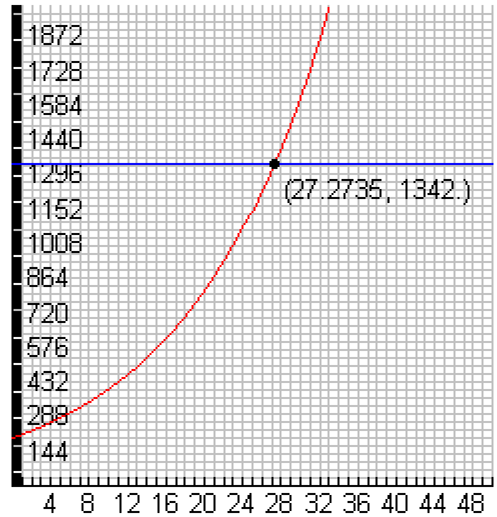
$$A = 778.25(1+\frac{.132}{12})^{(12)(10)} = \$2,892.40$$

Finding the time period of an investment

EX. Jackie has $200 to invest at 7% annual interest compounded monthly. How long will it take for her investment to grow to $1,342?

$$1342 = 200(1+\frac{.07}{12})^{12t}$$

We can solve graphically: Graph $y_1 = 1342$ and $y_2 = 200(1+\frac{.07}{12})^{12t}$



The intersection is at the point (27.27, 1342), therefore it will take 27 years 3 months to have \$1342 in the bank.

We can solve algebraically,

$$1342 = 200(1+\frac{.07}{12})^{12t} \rightarrow \frac{1342}{200} = (1+\frac{.07}{12})^{12t}$$

$$\rightarrow \ln\frac{1342}{200} = \ln(1+\frac{.07}{12})^{12t} \rightarrow \ln\frac{1342}{200} = 12t\ln(1+\frac{.07}{12}) =$$

$$\frac{\ln\frac{1342}{200}}{12\ln(1+\frac{.07}{12})} = t$$

<u>Interest Compounded Continuously</u>

$A = Pe^{rt}$ r- interest rate t- time

Ex. Jessie and Jackie's brother Cole is a bit wiser with money. He has earned \$1500 over the summer mowing lawns. He finds a bank that compounds interest continuously at a rate of 11.5%. He decides to put his money in the bank until he graduates from high school so, he will have some money for college. After 3 years he graduates and is ready to start college. How much money does he have in the bank?

$$A = 1500e^{(.115)(3)} = \$2,117.98$$

Cole receives a track scholarship (at the U of U) and decides to keep the money in the bank until he graduates from college. After four years he graduates, how much money does he have in the bank now?

$$A = 1500e^{(.115)(7)} = \$3,355.04$$

Future Value of an Annuity

The future value FV of an annuity consisting of n equal periodic payments of R dollars at an interest rate *i* per compounding period (payment intervals) is:

$$FV = R\frac{(1+i)^n - 1}{i}$$

Ex. At the end of each quarter year Clay makes a payment of $100 into his college fund. It earns 8.11% annual interest compounded quarterly. What will be the value of his investment in 5 years when he is ready to start college?

$$FV = 100\frac{(1+\frac{.0811}{4})^{(5)(4)} - 1}{\frac{.0811}{4}} = \$2,436.40$$

Present Value of an Annuity

The present value of an annuity consisting of n equal payments of R dollars earning an interest rate *i* per period is:

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

EX. After their kids finally graduate from college the Lambournes can finally buy a new vehicle. They purchase an SUV for $33,333. What are the monthly payments for a 5 year loan with a $3,000 down payment and an APR of 3.3%?

$$30,333 = R\frac{1-(1+\frac{.033}{12})^{(-5)(12)}}{\frac{.033}{12}}$$

R = $549.10